New methods for computing regularity of wavelets and subdivisions

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Abstract

It is well known that the convergence and regularity of subdivisions are expressed in terms of the joint spectral characteristics of suitable matrices (transition matrices). The joint spectral radius (Rota, Strang, 1960) is responsible for convergence and regularity in the spaces C^k, the p-radius (Lau, Wang, 1995) is in the spaces L_p, $p \in [1, +\infty)$ and in the Sobolev spaces W_p^k, the Lyapunov exponent (Furstenberg, Kesten, 1960) indicates the local regularity almost everywhere, while the lower spectral radius (Gurvits, 1995) expresses the maximal local regularity. For one matrix, all those characteristics become the (usual) spectral radius. For a set of matrices, they are not expressed by their eigenvalues and are notoriously hard to compute or estimate, apart from a few exceptions (p-radius for even integers p, which is computed by an explicit formula). The evaluation of the joint spectral radius is NP-hard; for the Lyapunov exponent, this problem is algorithmically undecidable. Nevertheless, in recent works we came up with several methods that either provide sharp approximations for the joint spectral characteristics (for the Lyapunov exponent, for the p-radius) or find them precisely (for the joint spectral radius) in relatively high dimensions. The methods are based on analyzing invariant Lyapunov functions of matrix families. Such functions are either convex ("invariant norms") or concave ("invariant antynorms"). This is done by applying modern tools of convex analysis and of convex optimization.

As applications, we compute exact values of Hölder exponents for the limit functions of the butterfly subdivision schemes, of the Daubechies wavelet functions ψ_n of order up to $n = 20$, the local regularities of some popular refinable functions, etc. Some of the presented results are made in recent joint works with N.Guglielmi and R.Jungers.

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