NURPS Splines for Modelling and Simulation

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Abstract

Isogeometric Analysis (IgA) is a simulation paradigm aiming to reduce the gap between the worlds of Finite Element Analysis (FEA) and Computer-Aided Design (CAD). The main idea is to use the CAD representations not only to model physical domains but also to approximate the solution of differential problems. Tensor-product B-splines and Non-Uniform Rational B-Splines (NURBS) are common tools in CAD, and so they are in IgA. Unfortunately, the tensor-product structure precludes a strictly localized refinement. This motivates the interest in alternative spline structures for IgA.

In this talk we discuss the use of quadratic Powell-Sabin (PS) splines in the context of IgA [1, 3, 4]. These splines are defined on triangulations endowed with a particular macro-structure and can be represented with basis functions possessing properties similar to the classical (tensor-product) B-splines. A rational extension, the so-called Non-Uniform Rational PS (NURPS) splines, can also be easily defined. They allow an exact representation of quadrics, and their shape can be locally controlled by control points and weights in a geometrically intuitive way. Higher order B-splines on triangulations with PS macro-structure can be found in [2].

Thanks to their structure based on triangulations, NURPS splines offer the flexibility of classical finite elements with respect to local mesh refinement and are not confined to quadrilateral parametric domains. Moreover, they share with tensor-product NURBS the increased smoothness and the B-spline-like basis. Hence, they constitute a natural bridge between classical FEA and NURBS-based IgA. We will illustrate the use of PS/NURPS splines in IgA with several numerical examples.

References

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